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A perturbative analysis of tachyon condensation

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Abstract: Tachyon condensation in the open bosonic string is analyzed using a perturbative expansion of the tachyon potential around the unstable D25-brane vacuum. Using the leading terms in the tachyon potential, Padé approximants can apparently give the energy of the stable vacuum to arbitrarily good accuracy. Level-truncation approximations up to level 10 for the coefficients in the tachyon potential are extrapolated to higher levels and used to find approximants for the full potential. At level 14 and above, the resulting approximants give an energy less than -1 in units of the D25-brane tension, in agreement with recent level-truncation results by Gaiotto and Rastelli. The extrapolated energy continues to decrease below -1 until reaching a minimum near level 26, after which the energy turns around and begins to approach -1 from below. Within the accuracy of this method, these results are completely consistent with an energy which approaches -1 as the level of truncation is taken to be arbitrarily large.

Keywords: Bosonic Strings, D-branes, Spontaneous Symmetry Breaking.

Contents

1. Background

Much effort has been expended over the last few years to confirm Sen's conjectures [1] on tachyon condensation in the open bosonic string (for reviews see [2]). Sen suggested that the unstable open string vacuum in 26 dimensions should be understood as containing a space-filling D25-brane. Sen's first conjecture states that the open bosonic string has a nontrivial locally stable vacuum with energy $-T_{25}V$ relative to the unstable perturbative open string vacuum, where T_{25} is the tension of the D25-brane, and V is the volume of spacetime; this new stable vacuum should correspond to the true closed string vacuum with no D-branes. This conjecture has been tested by several authors using the approach of truncating Witten's open string field theory [3] to include only fields below a certain total oscillator level. In [4] Kostelecky and Samuel used level truncation up to level 4, including 10 fields. These authors found that at low levels of truncation Witten's theory has a stable vacuum solution, and that the energy of this stable vacuum seems to converge as the level of truncation of the theory is increased. At that time, however, D-branes had not yet been understood, and there was no natural way to interpret the energy of this new vacuum. In [5] Sen and Zwiebach reconsidered this calculation in the light of Sen's conjectures. They found that at level 4, the vacuum energy of the nontrivial solution of the level-truncated theory is 0.986 times the energy $-E_0 = -T_{25}V$ predicted by Sen. In [6], Moeller and Taylor extended this calculation to higher levels. We found that as the level of truncation is increased, the vacuum energy continues to decrease, until at level 10 the vacuum energy reaches $-0.9991\,E_0$. These calculations seemed to indicate that level truncation gives a systematic approximation to the full string field theory, and that the vacuum energy computed in level truncation would converge monotonically to $-E_0$. The accuracy of this picture was thrown into doubt when Gaiotto and Rastelli recently announced [7] that when the level of truncation is increased further, the vacuum energy *overshoots* $-E_0$. They found that at level 14 the vacuum energy becomes $-1.0002\,E_0$, and that the energy continues to decrease monotonically until at level 18 the vacuum energy is $-1.0005\,E_0$.

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In this paper we use an alternative approach based on perturbation theory to estimate the energy of the nontrivial vacuum of open string field theory. We use level truncation to determine the values of the coefficients in a perturbative expansion of the effective zero-momentum tachyon potential around the usual open string vacuum. We then extrapolate these values to higher levels, and use the actual and predicted values of the coefficients at various levels to compute Padé approximants for the effective potential. The resulting Padé approximants seem to give highly reliable estimates for the vacuum energy. Furthermore, the Padé approximants based on the extrapolated perturbative coefficients give rise to energies which closely match those found by Gaiotto and Rastelli. After overshooting the desired value of $-E_0$ for the energy, however, the predicted energies turn around again near level 26, eventually returning close to the desired value. These calculations show that the results of Gaiotto and Rastelli are fully compatible both with earlier computations of the coefficients of the effective tachyon potential and with the conjecture that the energy of the nontrivial vacuum in level-truncated SFT asymptotically approaches the value predicted by Sen as the level is taken arbitrarily large.

In this paper we work exclusively in Feynman-Siegel gauge. Other choices of gauge were explored in [8]; for some other gauge choices, the level-truncated approximations to the energy were found to go below $-E_0$ (sometimes at levels as low as $L = 4$), and to be non-monotonic. The results of this paper suggest that this is in fact the generic behavior of level-truncated approximations in any gauge; we expect that applying the methods of this paper to the calculation in other gauges will give similar results.

In section 2 we discuss the calculation of coefficients in the effective tachyon potential. In section 3 we describe the method of Padé approximants and their use in studying the vacuum energy of SFT. In section 4 we use Padé approximants on the coefficients computed in section 2 and summarize our results.

2. Perturbative tachyon potential

The effective potential for the $p = 0$ tachyon field ϕ is given by

$$\begin{aligned} V(\phi) &= \sum_{n=2}^{\infty} c_n (\kappa g)^{n-2} \phi^n \\ &= -\frac{1}{2}\phi^2 + (\kappa g)\phi^3 + c_4(\kappa g)^2\phi^4 + c_5(\kappa g)^3 + \cdots , \end{aligned} \tag{2.1}$$

where $\kappa = 3^{7/2}/2^7 \approx 0.365$ and where $c_4, c_5, \ldots$ are numerical constants. Closed form expressions for these coefficients were given in [9]–[11]. These expressions cannot be evaluated exactly, however, and must be approximated numerically. Level truncation of Witten's string field theory gives an efficient means of approximating these coefficients [4, 6, 11]. In [6], we computed the coefficients up to c_{60} at truncation levels up to (10, 20), meaning that all fields up to level 10 and interactions up to total level 20 were included.

For example, for c_4 successive level-truncation approximations give

$$\begin{aligned} c_4^{[2]} &= -\frac{34}{27} \approx -1.259259 \\ c_4^{[4]} &\approx -1.472489 \\ c_4^{[6]} &\approx -1.556198 \\ c_4^{[8]} &\approx -1.600425 \\ c_4^{[10]} &\approx -1.627694\,, \end{aligned}$$

where by $c_4^{[L]}$ we denote the approximation to c_4 in level approximation $(L, 2L)$. Note that throughout this paper we systematically use level truncations $(L, 2L)$; empirical observations indicate that level $(L, 2L)$ approximations are generally very close to level $(L, 3L)$ approximations, although the former are easier to compute [6, 7]. An analytic expression for c_4 given in [9] was numerically evaluated in [4, 11] and found to give

$$c_4 \approx -1.742 \pm 0.001\,. \tag{2.2}$$

In [11], approximations to c_4 were computed using oscillator level truncation of an exact expression written in terms of an infinite matrix of Neumann coefficients. These approximations $c_4^{(L)}$ were computed up to oscillator level $L = 100$, and were found to behave at large L as

$$c_4^{(L)} \approx -1.742 + 0.80\, L^{-1} + \mathcal{O}(L^{-2})\,. \tag{2.3}$$

To use the Padé approximant method described in the next section, we need an accurate approximation to the coefficients c_n for n up to 26 or so. While the method of [11] is much more efficient for computing c_n for small n at high levels, at large values of n this approach becomes more difficult as the number of diagrams which must be summed increases quickly with n. Thus, in this paper we will use the approximate coefficients $c_n^{[L]}$ computed in [6] up to level $L = 10$. From the analysis of [11], however, we expect that these approximate coefficients will have corrections which can be expressed as a power series in $1/L$. This allows us to use the data for $L \leq 10$ to predict the values of the approximate coefficients for $L > 10$. For example, using the values of $c_4^{[6]}, c_4^{[8]}, c_4^{[10]}$ from (2.2) we can fit to a function of the form $a + b/L + c/L^2$ to get

$$c_4^{\{2,L\}} \approx -1.74226 + 0.59482\, L^{-1} - 0.10987\, L^{-2}\,. \tag{2.4}$$

Using the value of $c_4^{[4]}$ in addition, and allowing a term of order L^{-3} gives

$$c_4^{\{3,L\}} \approx -1.74204 + 0.59213\, L^{-1} - 0.09933\, L^{-2} - 0.01346\, L^{-3}\,. \tag{2.5}$$

Including $c_4^{[2]}$ and a term of order L^{-4} gives

$$c_4^{\{4,L\}} \approx -1.74190 + 0.59013\, L^{-1} - 0.08916\, L^{-2} - 0.03550\, L^{-3} + 0.01718\, L^{-4}\,. \tag{2.6}$$

The low order terms in these extrapolations seem to be converging well, so we expect that (2.4)–(2.6) give increasingly good estimates of the level-truncated approximations to the coefficient c_4. We have performed similar extrapolations for the higher coefficients c_n with similar results. We will use these extrapolations in Section 4 to estimate the value of the vacuum energy in various level-truncated approximations.

3. Padé approximants

The power series expansion (2.1) of the tachyon effective potential $V(\phi)$ has a finite radius of convergence of approximately $\phi_r \approx 0.25/g$ [6]. This finite radius of convergence arises due to the breakdown of Feynman-Siegel gauge when ϕ approaches the critical value $\phi_c \approx -0.25/g$ [8]. At this point the function $V(\phi)$ has a square root branch cut singularity. The nontrivial vacuum occurs near $\phi \approx 1/g$, so that the series expansion of $V(\phi)$ is badly divergent near the true vacuum. Thus, we cannot directly study the local minimum of the tachyon effective potential by summing the perturbation series (2.1).

A powerful method for dealing with series expansions outside their radius of convergence is given by the method of Padé approximants. Given a power series

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots , \tag{3.1}$$

the Padé approximant $P_N^M(x)$ is the unique rational function with numerator of degree M and denominator of degree N which agrees with the power series expansion (3.1) out to order $M + N$

$$P_N^M(x) = \frac{\sum_{i=0}^M b_i x^i}{\sum_{j=0}^N d_j x^j} = \sum_{k=0}^{N+M} a_k x^k + \mathcal{O}\left(x^{N+M+1}\right). \tag{3.2}$$

While the partial sums computed by truncating (2.1) at finite order do not converge near the stable tachyon vacuum, these partial sums may be used to compute Padé approximants with much better convergence behavior. For example, consider the approximate perturbative potential given by truncating (2.1) at ϕ^K and using the level 2 approximations $c_n^{[2]}$ to the coefficients. For $K = 4$, the resulting quartic

$$-\frac{1}{2}\phi^2 + \kappa g \phi^3 - \frac{34}{27}(\kappa g)\phi^4 \tag{3.3}$$

has no local minima, while the Padé approximant

$$P_1^3(\phi) = \frac{-\frac{1}{2}\phi^2 + \frac{10}{27}\kappa g \phi^3}{1 + \frac{34}{27}\kappa g \phi} \tag{3.4}$$

does.

We have investigated the behavior of various families of Padé approximants using the coefficients of the tachyon effective potential computed at low truncation levels. We find that the family of approximants P_n^{n+2} does very well at reproducing the effective potential out to the nontrivial vacuum. For example, at level 4 the effective potential has a minimum with energy $-0.986403\, E_0$ [5, 6]. Using Padé approximants on the level 4 coefficients we find that the Padé approximants P_n^{n+2} give increasingly accurate estimates of vacuum energy, giving for example (in units of E_0)

$$\begin{aligned} E_{10}^{12} &= -0.9864053 \\ E_{11}^{13} &= -0.9864036 \\ E_{12}^{14} &= -0.9864033\,. \end{aligned} \tag{3.5}$$

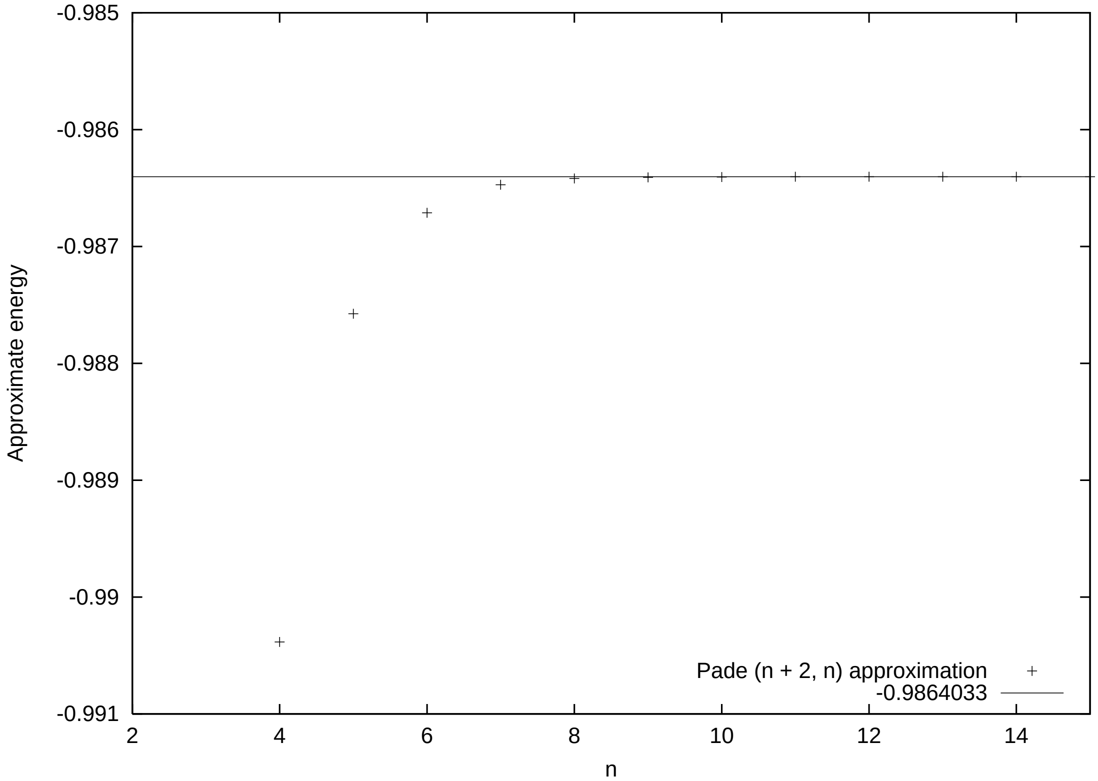

Figure 1: Level 4 vacuum energy using Padé approximants.

The Padé approximants converge up to the limit of numerical precision possible from the coefficients we have used. The energies arising from using Padé approximants P_n^{n+2} to estimate the vacuum energy at level 4 are graphed in figure 1. Performing a similar analysis on the approximate coefficients at other levels of truncation, we find that up to level 10, the Padé approximant P_{12}^{14} comes within 10^{-6} of the exact value of the energy at the minimum of the approximate effective potential. We expect similar behavior at higher levels. We use this approximant in our further analysis.

4. Estimating the vacuum energy

In section 2 we described a method for estimating the coefficients $c_n^{[L]}$ which would be computed using level truncation at levels $L > 10$. In section 3 we described a method for using coefficients in the effective tachyon potential to estimate the vacuum energy. We can now combine these approaches to estimate the vacuum energy which would be computed in a level L truncation with $L > 10$.

We have used the Padé approximant P_{12}^{14} to estimate the vacuum energy which would arise from the tachyon potential given by each of the three families of extrapolated coefficients $c_n^{\{m,L\}}$ given in (2.4)–(2.6) for $m = 2, 3, 4$. The resulting vacuum energies are graphed in figure 2 and compared to the exact results of Gaiotto and Rastelli at levels 12-16 [7]. As can be seen from the graph, while the order $1/L^2$ extrapolation of the perturbative coeffi-

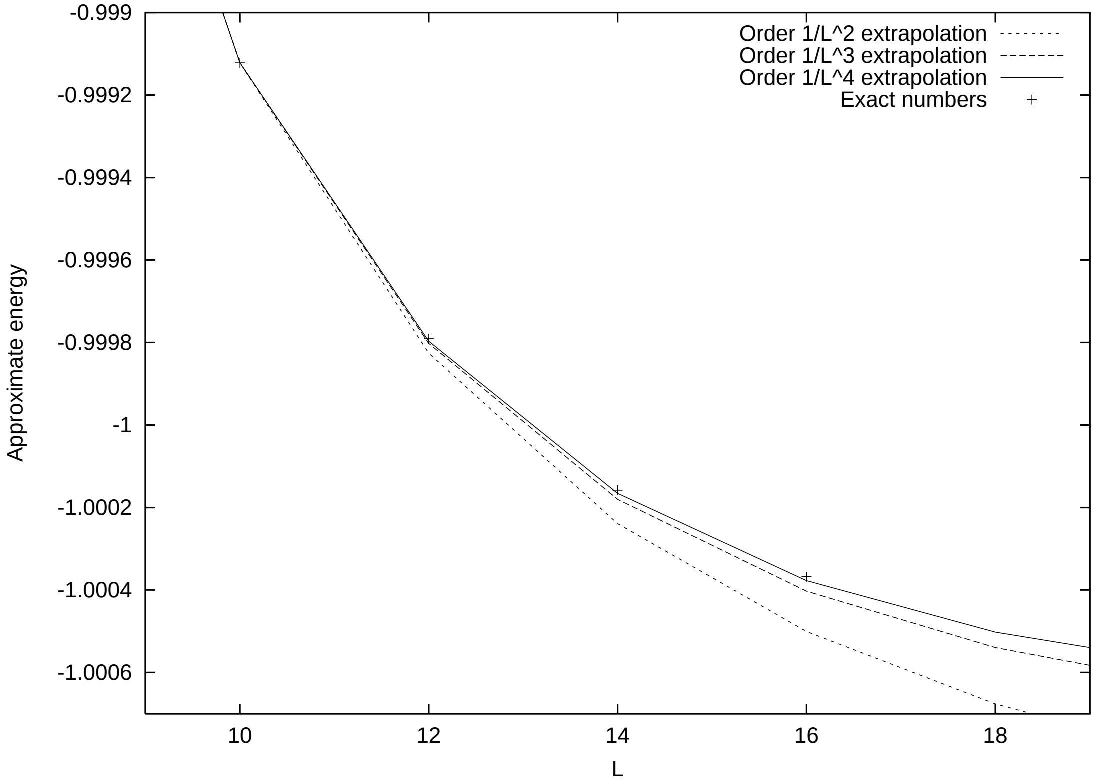

Figure 2: Extrapolating vacuum energy in level truncation $L > 10$.

cients gives an energy which overshoots the correct values, the order $1/L^4$ extrapolation is very close to the exact values. For example, at level 16 Gaiotto and Rastelli found

$$E^{[16]} = -1.0003678\,. \tag{4.1}$$

The order $1/L^4$ extrapolation gives

$$E^{\{4,16\}} \approx -1.0003773 \tag{4.2}$$

while the order $1/L^3$ and $1/L^2$ extrapolations give $E^{\{3,16\}} \approx -1.0004030$ and $E^{\{2,16\}} \approx -1.0005007$ respectively.

Seeing that we can fairly accurately reproduce the values found by Gaiotto and Rastelli for the energy in the level-truncated theory at levels $L > 10$, it is of great interest to extrapolate to still higher values of L. We have done this for each of the three extrapolations we are using. In each case, we find that at a particular level the extrapolated energy reaches a minimum value and then starts to climb back towards -1. Table 1 summarizes for each of the extrapolations we are using the level at which the extrapolated energy reaches a minimum, the energy at the corresponding minimum, and the energy as $L \to \infty$

The extrapolated energy is graphed as a function of $\ln L$ in figure 3.

It is clear that these results are compatible both with the observation of Gaiotto and Rastelli that the energy shoots below -1 at level 14 and with the conjecture that as

Order	L at minimum E	E_{min}	E_{∞}
$1/L^2$	138	-1.001247	-1.001239
$1/L^3$	34	-1.000761	-1.000444
$1/L^4$	28	-1.000661	-1.000159

Table 1: Minimum and asymptotic energies for different extrapolations.

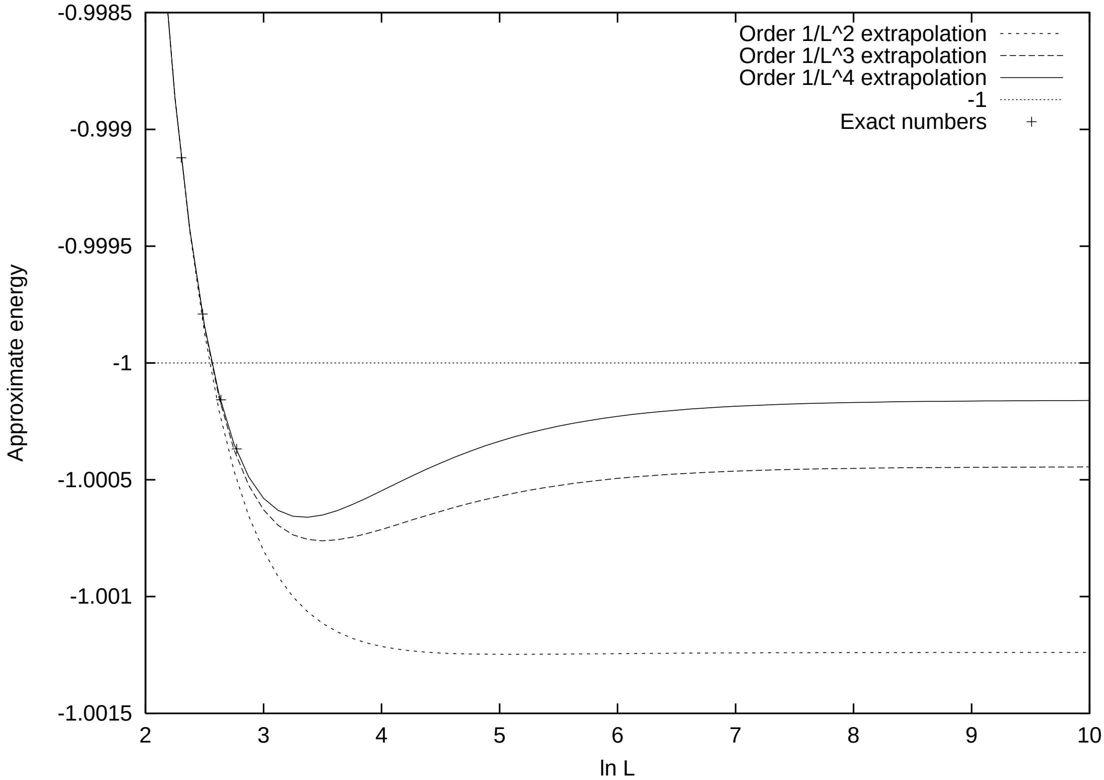

Figure 3: Extrapolated vacuum energy for large L.

$L \to \infty$ the energy approaches -1 in a controlled fashion. It would be nice to have a clearer theoretical understanding of why the individual coefficients in the perturbative expansion of the tachyon effective potential have corrections described by a power series in $1/L$ in level truncation, as this feature was crucial in getting the computations in this paper to work out.

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